Possibility and probability: application examples and comparison of two different approaches to uncertainty evaluation

Possibilité et probabilité : exemples d'application et comparaison entre deux approches à l'évaluation de l'incertitude de mesure

[1]Alessandro FERRERO, [2]Jean-Michel POU, [1]Marco PRIOLI, [1]Simona SALICONE
[1]Politecnico di Milano, Dipartimento di Elettronica, Informazione e Bioingegneria, [2]Delta Mu
[1]Italy, [2]France

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Abstract

During the last decades a new mathematical model for processing incomplete information has been developed by Shafer [1] and Dempster [2]. Their theory of evidence is a strict mathematical generalization of the well-known probability theory and the more recent fuzzy logic theory proposed by Zadeh [3]. Both theories can be derived by the theory of evidence as particular cases and Zadeh's theory has been encompassed, in this framework, in the so called possibility theory.

As a generalization of probability, evidence and its other particular case of possibility, have attracted the attention of metrologists, since they appear to be a promising tool for overcoming some limitations of the present GUM approach, fully based on probability, in evaluating and expressing measurement uncertainty [4,5].

One of the strongest underlying constraints of the GUM approach [6] is that all contributions to uncertainty have to be generated by random effects, in order to be properly modelled and processed in probabilistic terms. Indeed, a strong assumption of the GUM is that *the result of a measurement has been corrected for all recognized significant systematic effects and that every effort has been made to identify such effects* [6].

While this is surely the best practice in primary metrology and in calibration labs, there are many practical industrial situations where, mainly due to cost reasons, systematic effects can be only partially recognized, usually by identifying a possible variation range, but corrections are not applied. Nevertheless, there is still interest in evaluating their contribution to uncertainty, and how they propagate through the whole measurement process. An example of an important systematic effect that cannot be always compensated for in a cost effective way is ambient temperature.

Actually, the only way to assess whether a systematic effect is significant or not for a given measurement process is to evaluate whether its contribution to uncertainty leads to exceed the desired target uncertainty or not. In the first case it has to be corrected, whilst in the latter it can be tolerated.

Recent studies have proved that the uncertainty contributions due to uncompensated and partially unrecognized systematic effects can be effectively modeled and processed inside the possibility theory, where measurement results can be expressed, together with their associated uncertainty, in terms of Random-Fuzzy Variables [7-9].

The final paper will briefly recall the theoretical fundamentals of this new mathematical approach and will propose a couple of simple, though significant examples, in the field of electrical measurement and in the field of dimensional metrology, of how uncertainty contributions due to uncompensated systematic effects caused by temperature variations can be represented and processed.

References

[1] G. Shafer, *A Mathematical Theory of Evidence.* Princeton, NJ, USA: Princeton Univ. Press, 1976.

[2] A. P. Dempster, «Upper and lower probabilities induced by a multivalued mapping», *The Annals of Mathematical Statistics* **38** (2), pp. 325–339, 1967.

[3] L. A. Zadeh, "Fuzzy logic and approximate reasoning," *Synthese*, vol. 30, n. 3/4, pp. 407–428, Sept. 1975.

[4] G. Mauris, L. Berrah, L. Foulloy, A. Haurat, "Fuzzy handling of measurement errors in instrumentation," *IEEE Trans. Instr. Meas.*, vol. 49, no. 1, pp. 89–93, 2000.

[5] A. Ferrero, S. Salicone, "Uncertainty: Only one mathematical approach to its evaluation and expression?", *IEEE Trans. Instr. Meas.*, vol. 61, n. 8, pp. 2167-2178, 2012.

[6] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP and OIML, Evaluation of Measurement Data—Guide to the Expression of Uncertainty in Measurement, 2008. JCGM 100:2008.

[7] A. Ferrero, S. Salicone, "The random-fuzzy variables: A new approach for the expression of uncertainty in measurement," *IEEE Trans. Instr. Meas.*, vol. 53, no. 5, pp. 1370–1377, 2004.

[8] A. Ferrero, S. Salicone, "The theory of evidence for the expression of uncertainty in measurement," in P*roc. 12 Int. Metrology Congress*, Lyon, France, Jun. 20_23, 2005.

[9] A. Ferrero, M. Prioli, S. Salicone, "Processing dependent systematic contributions to measurement uncertainty", *IEEE Trans. Instr. Meas.*, vol. 62, n.4, pp. 720-731, 2013.